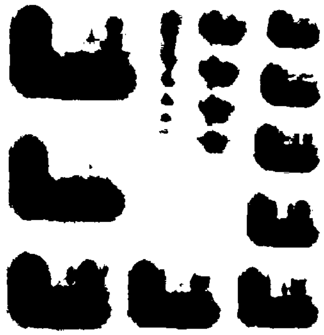
A153 247 NUMERICAL METHODS FOR STIFF ORDINARY AND ELLIPTIC PARTIAL DIFFENENTIAL EQ..(U) IBM THOMAS J WATSON RESEARCH CENTER YORKTOWN HEIGHTS NY L WERNER ET AL. 1/1

LASSIFIED FEB 85 AFOSR-TR-85-0336 F49620-83-C-0005 F/G 12/1 NL

END
FILMED
DTIC



(6)

NUMERICAL METHODS FOR STIFF ORDINARY AND ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

Final Report

I. SUMMARY OF PROGRESS

1. Wave-form (WF) relaxation methods for large-scale circuit analysis

Work has been completed or has progressed on items 2b, 2c, and 2e of Section IV of the Contract Proposal. More specifically: The convergence of the discretized version of the WR algorithm was shown [1] under suitable assumptions on the stability of the multistep methods employed and on the strength of the feedback (item 2b). Also, the effects of various decompositions of the large-scale circuit were discussed [1,2] (item 2e). In particular, in [1], a new decomposition which uses only neighboring circuits was shown to be effective for a large class of digital circuits. This decomposition, called ϵ-scheduling, dramatically decreases the storage requirements of the WR method. Moreover, since different subcircuits are usually integrated using different time steps, the effect of interpolation on the convergence of the WR method was studied (item 2c). Various sufficient conditions for the WR convergence were derived which combine accuracy requirements with some sort of stability condition, e.g. that the interpolation operator be of positive type, or be "passive", i.e. act boundedly on exponentially increasing sequences.

The above theoretical results have been used in a program, for hierarchical analysis of large scale MOSFET circuits, which implements an efficient scheduling (partitioning) algorithm (item 3e) and uses the charge as a state variable, leading to better numerical stability. The program is described in reference [3].

2. Fast direct methods for elliptic partial differential equations

Work has been completed or has progressed on items 3a-3d and 3f of Section IV of the Contract Proposal. More specifically: A one-parameter family of factored complex discretizations of the Laplace operator was derived [4]. These discretizations are second order accurate and serve as a basis for fast direct or iterative Poisson solvers on general two-dimensional regions (item 3a). The stability of these discretizations with respect to the propagation of rounding errors was analyzed [4]. A variant of the marching method, based on these factored discretizations, was proposed [4] which is much more stable than the conventional marching method and is thus applicable to grids with large numbers of discretizations steps in each direction, without resorting to domain decomposition or multiple shooting. The gain in stability is achieved by solving initial value problems for two first-order difference equations, rather than one second-order equation. Furthermore, the stability and accuracy of the method can be controlled by the choice of the above mentioned parameter (item 3b). The new version of the marching method was implemented and successfully tested [4] on a square region as a function of the number of discretization steps (item 3c), and on several examples of general regions with piecewise rectilinear boundaries, including multiply connected ones (item 3d). The numerical results confirmed the theoretical accuracy and stability predictions. The marching method was extended to the case of general mixed linear boundary conditions (item 3f), and the stability analysis was generalized to the use of rectangular (rather than square) discretization meshes.

II. PROPOSED RESEARCH

During the second year of the contract period, we propose to:

1a) Investigate the effects of stiffness on the convergence of the WR method, for both continuous systems and their discretizations.

1b) Devise algorithms for detecting strong feedbacks in large circuits and strategies for "windowing" WR in the presence of such feedbacks, with the object of optimizing the over-all efficiency of the method.

1c) Carry out numerical testing of the WR method.

2) Investigate the short-channel effects in MOS transistors by both analytical asymptotic methods and by numerical integration of the partial differential equations governing and typical semi-conductor devices.

3a) Study the use of the pre-conditioned conjugate gradient (PCG) method, in conjunction with the factored consistent complex discretizations of the Laplacian mentioned above, for the iterative solution of Poisson's equation on general two-dimensional regions.

3b) Apply the PCG method to approximate factored discretizations of the Laplacian, with a view toward stabilizing the computation and deriving iterative algorithms which are faster than the direct algorithm described above.

References

[1] F. Odeh, A. Ruehli and C. Carlin, "Robustness aspects of an adaptive waveform relaxation scheme", IEEE International Conference on Computer Design; ICCD 83, New York.

[2] C. Carlin, A. Ruehli and R. Odeh, "The waveform relaxation method for large scale circuit analysis", Proc. Sixth European Conference on Circuit Theory and Design; ECCTD 83, Stuttgart.

[3] J. Beetem et. al., "Large scale MOSFET circuit analyzer based on wave-form relaxation", (To be published).

[4] W. Liniger, "On factored discretizations of the Laplacian for the fast solution of Poisson's equation on general regions", IBM Report RC 10067, July 1983. To appear in BIT.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AD-A153247

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY: -	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 85-0336
6a. NAME OF PERFORMING ORGANIZATION: International Business Machines Corp. / 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code): Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights NY 10598	7b. ADDRESS (City, State and ZIP Code): Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR / 8b. OFFICE SYMBOL (If applicable): NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: F49620-83-C-0005
8c. ADDRESS (City, State and ZIP Code): Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS.: PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2304; TASK NO. A3; WORK UNIT NO.

11. TITLE (Include Security Classification): NUMERICAL METHODS FOR STIFF ORDINARY AND ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS, FINAL /SCIENTIFIC REPORT, AFOSR CONTRACT F49620-83-C-0005, /1 OCTOBER 1982 - 31 JANUARY 1985

12. PERSONAL AUTHOR(S): Werner Liniger and F. Odeh

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Final	FROM 1/10/82 TO 31/1/85	FEB 85	5

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The research under this effort was concerned with stable high-order methods for nonlinear stiff systems of ordinary differential equations, relaxation methods for large scale circuit analysis, and fast direct methods for elliptic partial differential equations on general regions. More specifically, the convergence of the discretized version of the wave-form relaxation algorithm was shown under suitable assumptions on the stability of the multistep methods employed and on the strength of the feedback. A new large-scale circuit decomposition was shown to be effective for a large class of digital circuits. In the area of fast direct methods for elliptic partial differential equations, a one parameter family of factored discretizations of the Laplace operator was derived. A variant of the marching method was proposed which is much more stable than the conventional approach and is thus applicable to grids with large numbers of discretization steps in each direction. Several journal publications and papers in conference proceedings appeared during the contract period. This research continues under AFOSR Contract F49620-85-C-0051.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
CPT John P. Thomas, Jr.	(202) 767- 5026	NM

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

END

FILMED

5-85

DTIC